Silly mistakes in Part A

Mia Clark-Webb et al.
mia.clark-webb@sjc.ox.ac.uk
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1. General advice

- If the question gives you an approximation, use it!
- Read the question carefully and think about what solution you want to obtain before starting complicated maths.

2. Thermal physics

- It is usually easier to use F than U for finding quantities like the equation of state, magnetic dipole moment, etc.
- $p = 2u/3$ for *any* non-relativistic gas.
- Don't forget to divide by $N!$ for the partition function of a gas whose particles are indistinguishable.
- Don't forget the chemical potential term in the energy of a gas in an open system:

$$U = -\frac{\partial \ln \mathcal{Z}}{\partial \beta} + \mu N. \tag{1}$$

3. Electromagnetism

- The magnetic field outside an object is not just the external field. Remember that the object changes the field as well.
- When finding the power transmission coefficient, compare the magnitudes of the Poynting vectors.

4. Quantum mechanics

- Don't forget to square when normalising a state, and don't forget to normalise in the first place.
- $0 \cdot 1$ is not 1 (for example, in $j(j+1)$).
- Don't forget how to take the determinant of a 3×3 matrix:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix} . \tag{2}$$

- Don't get the signs the wrong way round for perturbation theory. The signs in the denominator are always such that the second-order correction to the ground state energy is non-positive, and the order in the numerator is the other way round. That is,

$$
\begin{aligned}
\left|\delta\psi_0^{(1)}\right\rangle &= \sum_{n=1}^{\infty} \frac{\left\langle n\middle|\hat{H}\middle|0\right\rangle}{E_0 - E_n}|n\rangle, \\
\delta E_0^{(2)} &= \sum_{n=1}^{\infty} \frac{\left|\left\langle n\middle|\hat{H}\middle|0\right\rangle\right|^2}{E_0 - E_n} \leq 0.
\end{aligned} \tag{3}
$$

- Don't forget the $1/2m$ factor in the Hamiltonian. It should be $-\hbar^2/2m$.
- $\Delta x \Delta p$ is not $\leq \hbar/2$.

$$
\Delta x \Delta p \geq \frac{\hbar}{2}. \tag{4}
$$

- The ground state is not zero energy.
- For spin-symmetric fermions, the fact that $|00\rangle$ doesn't exist doesn't mean that there is no ground state. Instead, the ground state is $|01\rangle$ or $|10\rangle$.
- Remember the time dependence in the Heisenberg equation of motion:

$$
\frac{\mathrm{d}\hat{Q}}{\mathrm{d}t} = \frac{1}{\mathrm{i}\hbar}\left[\hat{Q}, \hat{H}\right] + \frac{\partial \hat{Q}}{\partial t}. \tag{5}
$$

5. Miscellaneous

- $\mathrm{e}^0 = 1$, not 0.
- In spherical coordinates, the contributors to the Jacobian are 1 for $\mathrm{d}r$, r for $\mathrm{d}\theta$, and $r \sin\theta$ for $\mathrm{d}\varphi$.
- In spherical coordinates, the divergence is not $\frac{\partial \boldsymbol{F}}{\partial r}$:

$$
\boldsymbol{\nabla} \cdot \boldsymbol{F} = \frac{1}{r^2}\frac{\partial(r^2 F_r)}{\partial r} + \ldots \text{ (just use the Data Sheet™, it's easier).} \tag{6}
$$